

**B. Tech Degree I & II Semester Examination in  
Marine Engineering, May 2008**

**MRE 102 ENGINEERING MATHEMATICS II**

Time : 3 Hours

Maximum Marks : 100

(All questions carry EQUAL marks)

- I. (a) Solve the system by Gauss elimination method.

$$5x + 2y + z = 29$$

$$x + 4y + 2z = 4$$

$$2x - y + 10z = 44$$

- (b) Obtain the eigen values and eigen vectors of the matrix

$$\begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$$

**OR**

- II. (a) State and prove the necessary and sufficient condition for a function  $f(z)$  to be analytic.

- (b) Evaluate  $\int_c \frac{3z^2 + z}{z^2 - 1} dz$  where  $c$  is the circle  $|z - 1| = 1$ .

- (c) Expand  $\frac{1}{z^2 - 3z + 2}$  in the region  $1 < |z| < 2$ .

- III. (a) Solve  $y - x \frac{dy}{dx} = a \left( y^2 + \frac{dy}{dx} \right)$

- (b) Solve  $x(1 - x^2) \frac{dy}{dx} + (2x^2 - 1)y = x^3$

- (c) Find the orthogonal trajectory of the cardioid  $r = a(1 - \cos \theta)$

**OR**

- IV. (a) Solve  $(D - 2)^2 y = 8e^{2x} + 8 \sin 2x + 8x^2$

- (b) Solve the simultaneous equations

$$\frac{dx}{dt} + 4x + 3y = t$$

$$\frac{dy}{dt} + 2x + 5y = e^t$$

- V. (a) Obtain the Fourier series for the function  $f(x)$  given by

$$f(x) = \begin{cases} 1 + \frac{2x}{\pi}, & -\pi \leq x \leq 0 \\ 1 - \frac{2x}{\pi}, & 0 \leq x \leq \pi \end{cases}$$

Deduce that  $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \dots = \frac{\pi^2}{8}$

(Turn Over)

(b) Prove that  $\beta(m, n) = \frac{\sqrt{m} \sqrt{n}}{\sqrt{m+n}}$

OR

- VI. (a) Obtain the half range sine series for  $e^x$  in  $0 < x < 1$ .  
 (b) Using Fourier sine integral, show that

$$\int_0^{\infty} \frac{1 - \cos \pi \lambda}{\lambda} \sin x \lambda d\lambda = \begin{cases} \frac{\pi}{2} & \text{when } 0 < x < \pi \\ 0 & \text{when } x > \pi \end{cases}$$

- VII. (a) Find the Laplace transform of  $\frac{e^{-at} - e^{-bt}}{t}$

- (b) Find the inverse Laplace transform of  $\frac{s^3}{s^4 - a^4}$

- (c) Evaluate  $L^{-1} \left\{ \frac{s}{(s^2 + a^2)^2} \right\}$  by Convolution theorem.

OR

- VIII. (a) Derive the Laplace transform of periodic function.

- (b) Solve by transform method  $\frac{d^2 y}{dt^2} - 3 \frac{dy}{dt} + 2y = 4t + e^{3t}$  where  $y(0) = 1$  and  $y'(0) = -1$

- IX. (a) State Baye's theorem.

- (b) A bag X contains 2 white and 3 red balls and a bag Y contains 4 white and 5 red balls. One ball is drawn at random from one of the bags and is found to be red. Find the probability that it was drawn from bag Y.

- (c) Find the mean and variance for the following distribution :

x	: 8	12	16	20	24
y(x)	: $\frac{1}{8}$	$\frac{1}{6}$	$\frac{3}{8}$	$\frac{1}{4}$	$\frac{1}{12}$

OR

- X. (a) If the joint pdf of x and y is  $f(x, y) = \frac{1}{8}(6 - x - y)$ ,  $0 < x < 2$   
 $2 < y < 4$

- (i) Find the marginal pdf of x and y  
 (ii) Test the independence of x and y  
 (iii) Find the conditional pdf of x given y and y given x.

- (b) If x and y are r.v's with joint pdf  $f(x, y) = \frac{x+2y}{18}$  where

$(x, y) = (1, 1), (1, 2), (2, 1), (2, 2)$  then find the marginal pdf's.  
 Also examine whether x and y are independent.

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